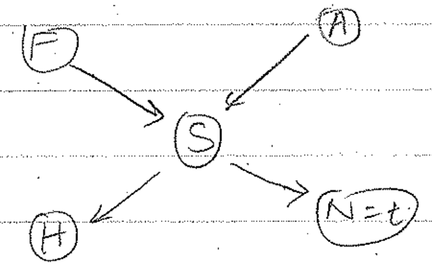
2/20/14

BN: VARIABLE ELIMINATION

1 Extending VE to MAP Inference



Compute: $\underset{f,a,s,h}{\operatorname{argmax}} \; P(F=f, A=a, S=s, H=h \mid N=t)$

Notice $\underset{f,a,s,h}{\operatorname{argmax}} \; P(f,a,s,h \mid N=t) = \underset{f,a,s,h}{\operatorname{argmax}} \; P(f,a,s,h,N=t)$

Let's focus on finding max value first. We'll find argmax next.

$$\underbrace{\max_f \max_a \max_s \max_h}\; P(f)P(a)P(s \mid f,a)P(h \mid s)P(N=t \mid s)$$

2^4 settings. Can we do better like the marginal case? Yes!

$$\boxed{\max_i \{a \cdot b_i\} = a \cdot \max_i \{b_i\}}$$

if $a \geq 0$ So let's use this

Elim Ordering: H, S, A, F

Iter	Current Factors	Elimination	New Factors
1	$P(F)\,P(A)\,P(S\mid F,A)\,P(H\mid S)\,P(N=t\mid S)$	$\max_h P(h\mid s)$	$g_1(S)$
2	$P(F)\,P(A)\,P(S\mid F,A)\,P(N=t\mid S)\,g_1(S)$	$\max_s P(N=t\mid S)\,g_1(S)\,P(S\mid f,a)$	$g_2(f,a)$
3	$P(F)\,P(A)\,g_2(f,a)$	$\max_a P(a)\,g_2(f,a)$	$g_3(f)$
4	$P(F)\,g_3(F)$	$\max_f P(f)\,g_3(f)$	Proportional to MAP value

Now, to find argmax, let's back track

We know $f^* = \arg\max_f P(f)\,g_3(f)$ ← what setting of f led to MAP value

Now $a^* = \arg\max_a P(a)\,g_2(f^*,a)$

what setting of a (consistent with MAP setting of F) lead to MAP value?

$s^* = \arg\max_s P(N=t\mid s)\,g_1(s)\,P(s\mid f^*,a^*)$

$h^* = \arg\max_h P(h\mid s^*)$

(2) Can we generalize VE further?
What else can VE (or dynamic programming) do?

Consider an abstract "operator" $\otimes$
and another " " $\oplus$

If $\oplus$ distributes over $\otimes$ we can save computation.
Meaning?

$$\text{If } a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$$

Then

$$(a \otimes b_1) \oplus (a \otimes b_2) \oplus \dots \oplus (a \otimes b_k)$$

$$= a \otimes [b_1 \oplus \dots \oplus b_k]$$

$K + (k-1)$ operations

$(k-1)+1$ operations $\Rightarrow$ Savings!

In our case $\otimes$ is "product"
$\oplus$ is "sum" for sum-product VE
$\oplus$ is "max" for max-product VE

What else?
How about $\oplus \equiv$ top-K elements

This will result in a VE alg that
returns top-K Most probable configurations