

# ECE 5424: Introduction to Machine Learning

## Topics:

- Expectation Maximization
  - For GMMs
  - For General Latent Model Learning

Readings: Barber 20.1-20.3

Stefan Lee  
Virginia Tech

# Project Poster

- Poster Presentation: **Best Project Prize!**
  - Dec 6th 1:30-3:30pm
  - Goodwin Hall Atrium
  - Print poster (or bunch of slides)
    - Fedex, Library, ECE support, CS support
  - Format:
    - Portrait, 2 feet (width) x 36 inches (height)
    - See <https://filebox.ece.vt.edu/~f16ece5424/project.html>
- Submit poster as PDF by Dec 6<sup>th</sup> 1:30pm
  - Makes up the final portion of your project grade

# How to Style a Scientific Poster

- Layout content consistently
  - top to bottom, left to right in columns is common
  - usually numbered headings



# Inorganic Biochemistry of Iron Proteins

Jared J. Heymann, Claire J. Parker Siburt, Katherine D. Weaver,  
and Alvin L. Crumbliss

Duke University – Department of Chemistry – Durham, NC



## Purpose:

To study iron protein  
biochemistry from the  
perspective of the iron  
Protein = Ligand

## The Iron Paradox

Iron is needed for nearly  
every living cell  
  
Iron is toxic and can  
produce reactive oxygen  
species & must be  
controlled

## Iron Abundance In Humans

45-55 mg/kg in humans  
70% in Red Blood Cells (Hemoglobin)  
0.1% in Transferrin  
  
However  
Turnover of transferrin iron is ~30 mg / 24 hours with 80% of  
this Fe being transported to the bone marrow for hemoglobin  
synthesis  
  
Bacteria can also target Tf as a source of iron

Proteins act as the  
**1<sup>st</sup> & 2<sup>nd</sup> coordination  
shell** of iron and can  
modulate the **kinetics**  
and **thermodynamics**  
of reaction.

## Techniques:

Spectroelectrochemistry  
UV-Visible Spectroscopy  
Fluorescence Spectroscopy  
Difference Spectroscopy  
Stopped-Flow Kinetics  
SUPREX

## TRANSFERRIN

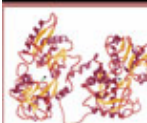
A mechanistic study of the  
iron release by receptor-bound  
transferrin using spectroelectrochemistry

## FERRIC BINDING PROTEIN

Role of a synergistic anion on modulating iron  
uptake in a bacterial transferrin by pathogenic  
bacteria: A study in kinetics and thermodynamics

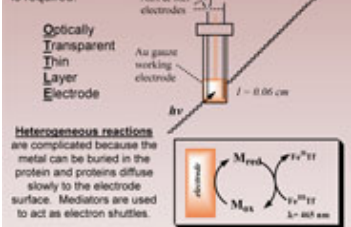
## HEMOGLOBIN

Effects of subunit cross-linking on  
hemoglobin oxidation states determined  
by spectroelectrochemistry



Transferrin

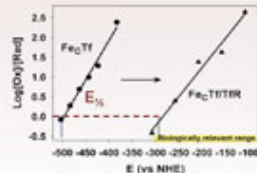
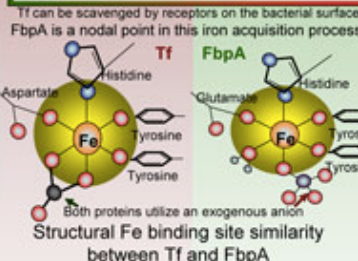
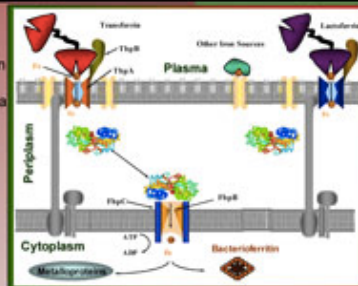
**Spectroelectrochemistry** utilizes a short pathlength  
created by an OTTE cell, to measure the variations in  
visible spectra as the analyte is oxidized or reduced by  
an externally applied potential. This technique is ideal for  
a biological analyte because only a small sample volume  
is required.



Iron loaded Tf binds to the human receptor and is  
taken into the cell by **endocytosis**. Tf releases  
iron inside in the endosome where the conditions  
are acidic (Andrews, 1999). However, the chemical  
mechanism is unclear. The reduction potential of  
Fe-Tf in the plasma (pH 7.4) and in the endosome  
(pH 5.8) is too low for biological reducing agents.



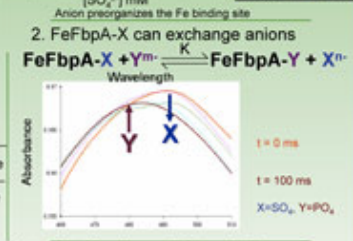
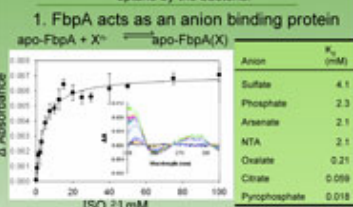
Andrews (1999) *N Engl J Med* 341, 1986-1993



Using spectroelectrochemistry, we measured a positive  
shift in redox potential of Fe-Tf upon receptor binding

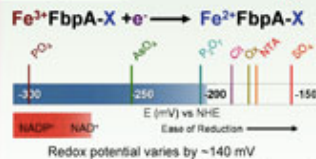
The transferrin receptor is capable of shifting the reduction potential into the range  
accessible by biological reducing agents, allowing for a redox mechanism of Fe release.  
Transferrin not only supplies iron to mammalian cells, but has been identified as a target  
for pathogens to mechanistically steal iron from their host.

Like Tf, BpA requires a synergistic anion to facilitate tight  
iron binding, which may play a role in ease and rate of Fe  
uptake by the bacteria.



3. Anion identity modulates both  
thermodynamic stability and redox potential

$\text{Fe}^{3+} + \text{BpA-X} \rightleftharpoons \text{Fe}^{3+}\text{BpA-X}$



Anion	log K
Phosphate	18.6
Aspartate	18.1
Oxalate	17.6
NTA	17.3
Pyrophosphate	17.3
Citrate	17.1
Sulfate	16.2

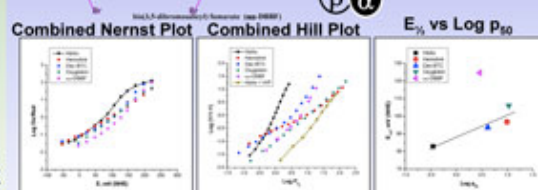
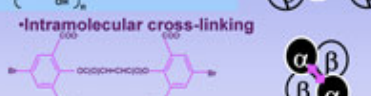
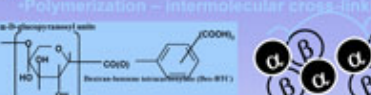
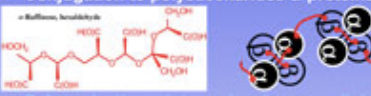
Thermodynamic stability varies by two orders of  
magnitude (14 kJ) based on identity of X

Iron transport can occur by a redox or non-redox mechanism in the periplasm. The  
thermodynamic stability and reduction potential are both varied by the identity of the  
synergistic anion. Kinetically labile exchange is possible in the diverse anionic  
conditions of the periplasm.

Heymann, Weaver, Metzner and Crumbliss (2006) unpublished.  
Dhungana, Anderson, Metzner and Crumbliss (2005) *Biochem* 44, 9606-18.  
Rouillac, Powell, Dhungana, Weaver, Metzner, Crumbliss and Fitzgerald (2004) *Biochem* 43, 15767-74.  
Dhungana, Taboy, Anderson, Vaughan, Aisen, Metzner and Crumbliss (2003) *PNAS* 100, 3659-64.

## Chemically modified Hb

- Pyridoxalation
- Pegylation
- Conjugation to polysaccharides & proteins



Sample	$E_{1/2}$ mV (NHE)	Oxidation $\mu_{10}$	Log $P_{10}$	Oxygenation $\mu_{10}$
HbA <sub>0</sub>	83	1.3	-0.455	2.28
Hemolys	97	0.7	0.994	0.71
Dex-BTC	94	0.9	0.618	1.49
OxylO <sub>2</sub> bin	106	0.9	1.028	1.11
aa-DBBF	125	1.0	0.461	1.56

## Implications

- Reengineering redox center not  
necessary
- Drive for autoxidation not  
thermodynamic
- Structural modifications perturb  
kinetics by altering exposure of  
heme cavity

## Modified Hb Conclusions

- Oxygen Transport**
- Loss of cooperativity
- Lower oxygen affinity
- T-state stabilization
- Anaerobic Reduction Potentials**
- Loss of cooperativity
- $E_{1/2}$  potential increased vs HbA<sub>0</sub>
- Normal physiological range
- Decreased tendency to form methHb

Bonaventura, Henkens, Weaver, Henrich, Pearce, Alayash and Crumbliss (2006) unpublished.  
Taboy, Bonaventura and Crumbliss (2002) *Meth. In Enzymology* 353, 187-209.  
Reiss (2001) *Chem. Rev.* 101, 2797-2919.

# How to Style a Scientific Poster

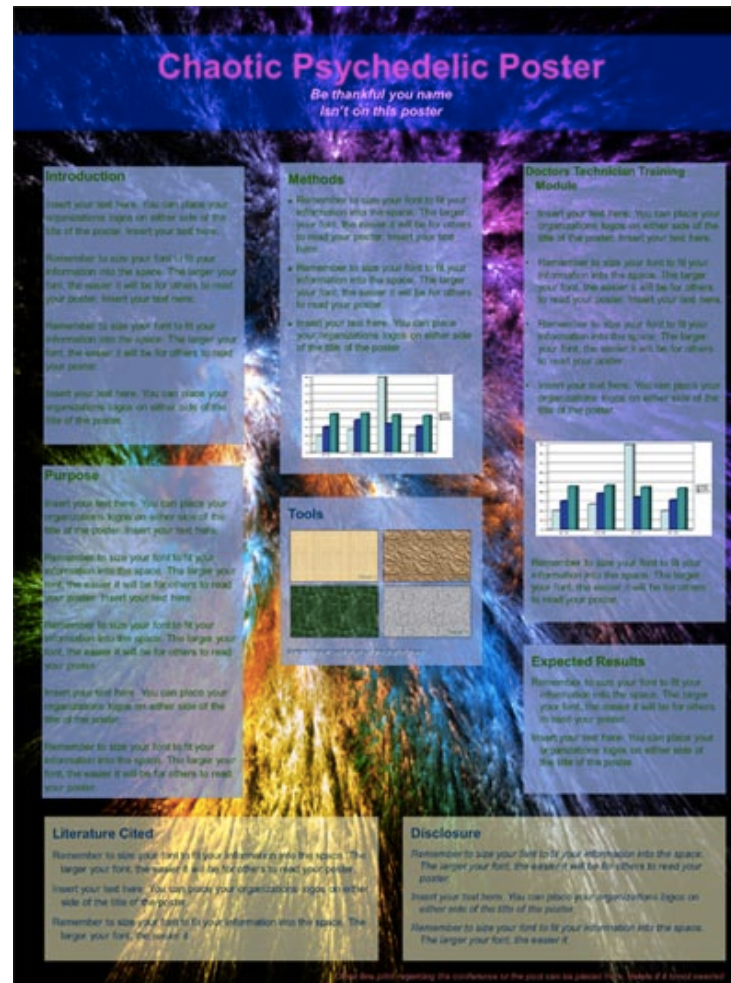
- Be cautious with colors

[illegible]



# How to Style a Scientific Poster

- Be cautious with colors



# How to Style a Scientific Poster

- Less text, more pictures. Bullets are your friend!

## I Can See!!!

Your name here, and names of others  
Place the name of your institution here

<h3>Abstract</h3> <p>Lorem ipsum dolor sit amet, consectetur adipiscing elit. Phasellus sapien nibh, rhoncus eu rutrum a, ultricies sit amet ligula. Ut luctus ultricies nulla tincidunt commodo. Proin a erat elit, sit amet lobortis mauris. Nunc sed sapien neque. Nulla cursus sem vitae magna mollis sollicitudin eget eu ante. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Vestibulum in mauris massa, nec vehicula lacus.</p>	<h3>Results</h3> 	<h3>Results</h3> 	<h3>Discussion</h3> <p>Lorem ipsum dolor sit amet, consectetur adipiscing elit. Phasellus sapien nibh, rhoncus eu rutrum a, ultricies sit amet ligula. Ut luctus ultricies nulla tincidunt commodo. Proin a erat elit, sit amet lobortis mauris. Nunc sed sapien neque. Nulla cursus sem vitae magna mollis sollicitudin eget eu ante. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Vestibulum in mauris massa, nec vehicula lacus. Suspendisse sagittis risus sit amet nunc fermentum porta. Donec sed magna nunc, a</p>
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<h3>Questions</h3> <p>Lorem ipsum dolor sit amet, consectetur adipiscing elit. Phasellus sapien nibh, rhoncus eu rutrum a, ultricies sit amet ligula. Ut luctus ultricies nulla tincidunt commodo. Proin a erat elit, sit amet lobortis mauris. Nunc sed sapien neque. Nulla cursus sem vitae magna mollis sollicitudin eget eu ante. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Vestibulum in mauris massa, nec vehicula lacus. Suspendisse sagittis risus sit amet nunc fermentum porta. Donec sed magna nunc, a dictum ipsum. Donec vitae neque mi, non dapibus arcu. Sed id mi gravida tortor hendrerit dapibus ac ut sapien. Nunc dignissim, tortor vitae facilisis tincidunt, libero</p>			<h3>References</h3> <p>Lorem ipsum dolor sit amet, consectetur adipiscing elit. Phasellus sapien nibh, rhoncus eu rutrum a, ultricies sit amet ligula. Ut luctus ultricies nulla tincidunt commodo. Proin a erat elit, sit amet lobortis mauris. Nunc sed sapien neque. Nulla cursus sem vitae magna mollis sollicitudin eget eu ante. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Vestibulum in mauris massa, nec vehicula lacus. Suspendisse sagittis risus sit amet nunc fermentum porta. Donec sed magna nunc, a dictum ipsum. Donec vitae neque mi, non dapibus arcu. Sed id mi gravida tortor hendrerit dapibus ac ut sapien. Nunc dignissim, tortor vitae facilisis tincidunt, libero arcu lacula eros, quis condimentum eros sem vitae risus. Donec justo eros, pretium vitae blandit at, viverra in lacus. Donec commodo scelerisque risa faucibus dapibus. Nulla facilisi. Aenean eget ligula eget mauris scelerisque facilisis a eget mi. Cras eget justo eget leo ultrices interdum. Nulla non quam ac augue condimentum pulvinar sed nec sapien. Nulla facilisi. Vestibulum semper cursus suscipit. Donec ornare fermentum sapien, dignissim mattis neque condimentum eu. In rhoncus vehicula tellus, molestie congue mi cursus eget. Suspendisse sit amet ligula in eros aliquet dignissim. Integer et mi quis dolor mollis rhoncus.</p>
<h3>Hypothesis</h3> <p>Lorem ipsum dolor sit amet, consectetur adipiscing elit. Phasellus sapien nibh, rhoncus eu rutrum a, ultricies sit amet ligula. Ut luctus ultricies nulla tincidunt commodo. Proin a erat elit, sit amet lobortis mauris. Nunc sed sapien neque. Nulla cursus sem vitae magna mollis sollicitudin eget eu ante. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Vestibulum in mauris massa, nec vehicula lacus. Suspendisse sagittis risus sit amet nunc fermentum porta. Donec sed magna nunc, a dictum ipsum. Donec vitae neque mi, non dapibus arcu. Sed id mi gravida tortor hendrerit dapibus ac ut sapien. Nunc dignissim, tortor vitae facilisis tincidunt, libero</p>			<h3>Acknowledgements</h3> <p>Lorem ipsum dolor sit amet, consectetur adipiscing elit. Phasellus sapien nibh, rhoncus eu rutrum a, ultricies sit amet ligula. Ut luctus ultricies nulla tincidunt commodo. Proin a erat elit, sit amet lobortis mauris. Nunc sed sapien neque. Nulla cursus sem vitae magna mollis sollicitudin eget eu ante. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Vestibulum in mauris massa, nec vehicula lacus. Suspendisse sagittis risus sit amet nunc fermentum porta. Donec sed magna nunc, a</p>

# How to Style a Scientific Poster

- Avoid giant tables
- Don't use more than 2 fonts
- Make sure everything is readable
- Make sure each figure has an easy to find take-away message (write it nearby, maybe in an associated box)



# How to Style a Scientific Poster

SCHOOL / DEPARTMENT / UNIT NAME HERE

# Title of the Research Study

Presenter name, Associates and Collaborators

## INTRODUCTION

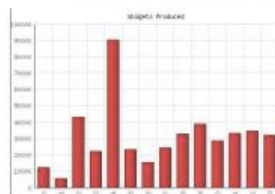
This editable template is in the most common poster size (48" x 36") and orientation (horizontally); check with the conference organizers for specific conference requirements regarding exact poster dimensions.

**Writing Style:**

The writing style for scientific posters should match the guidelines for the university. Use the Editorial Style Guide at <http://go.osu.edu/Vrg> for general guidance with academic titles, names of campus buildings, the correct way to refer to the campus, etc.

Copyright and Intellectual  
Property Guidelines

In today's world, just about everything is copyrighted, whether it carries the copyright symbol © or not. Moreover, under today's law, materials are protected by copyright as soon as they are completed. Copyright applies broadly to all creative pieces whether written on paper, sculpted in stone, found in cyberspace or created on videotape. Please visit <http://go.osu.edu/Vrh> for more information.



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fuiti no facitum apudm ai pnt. lxxvii de censu fuit nre volente  
nulli magis, quare ocl' alle ocl' hant i'it alt' p'ntes coemod' m  
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## AIM

### How to use this template

Highlight this text and replace it with new text from a Microsoft Word document or other text-editing program. The text size for body copy and headings and the typeface has been set for you. The text boxes and photo boxes may be resized, eliminated, or added as necessary. The references to the department, college and university, including the logo, should remain.

Head 3, to label the table below



Head 3, to label the table below



## METHODS

## Text

Be sure to spell check all text and have trusted colleagues proofread the poster. In general, authors should:

- Use the active tense
- Simplify text by using bullet points
- Use colored graphs and charts
- Use bold to provide emphasis; avoid capitals and underlining
- Avoid long numerical tables

Authors should re-write their paper so that it is suitable for the brevity of the poster format. Respect your audience. As a general rule, less is more. Use a generous amount of white space to separate elements and avoid data overkill. Refer to Web sites or other sources to provide a more in-depth understanding of the research.

## RESULTS

Images

Images must be 72 to 100 dpi in their final size, or use a rule of thumb of 2 to 4 megabytes of uncompressed .tif file per square foot of image. For instance, a 3x5 photo that will be 6x10 in size on the final poster should be scanned at 200 dpi.

We prefer that you import tiff or jpg images into PowerPoint. Generally, if you double click on an image to open it in Microsoft Photo Editor, and it tells you the image is too large, then it is too large for PowerPoint to handle too. We find that images 1200x1600 pixels or smaller work very well. Very large images may show on your screen but PowerPoint cannot print them.

Head 3, to label the table below



Head 3, to label the table below



### Preview

To see your in poster in actual size, go to view-zoom-100%. Posters to be printed at 200% need to be viewed at 200%.

### Printing and Laminating

CommTech Printing Services can print and laminate your research poster. To place your order, contact us by phone at 330-202-3508 or send an e-mail to [warren.119@csu.edu](mailto:warren.119@csu.edu). Plan ahead; allow at least seven business days for Printing Services to complete the order. Other dimensions are available; the charge is by square foot. Contact Printing Services for specific pricing information.

## CONCLUSIONS

We have created this template with scientific researchers in mind. We encourage any comments or suggestions so that we can continue to update and improve this template. E-mail [brown.3384@osu.edu](mailto:brown.3384@osu.edu) with suggestions.

## BIBLIOGRAPHY

1. Referentibus lux eorum adpositurum ducuntur tuerentur locorum  
etiamque hoc eorum iussu quoniam non innotuit illam  
quod iure verosimile sit et si te facillime modum in  
voluntate
2. Vel et vel element ad, consensum zzzzzzz et ego uto  
facile con esse enim zzzzzz et ducere sit, volubis si
3. Vel vendit, quod iustis amensum et con consumo et alio  
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etiamque non dolum amensum, enim

#### ACKNOWLEDGEMENTS

Check to make sure you've acknowledged partner and funding agencies, either with text or with their logos.



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COLLEGE OF FOOD, AGRICULTURAL,  
AND ENVIRONMENTAL SCIENCES

# How to Present a Scientific Poster

- Typical poster session at a conference:
  - You stand by your poster and people stop to check it out
- You will need:
  - 30 second summary
    - gives the visitor an overview and gauges interest
    - if they are not really interested, they will leave after this bit
  - 3-5 minutes full walkthrough
    - if someone stuck around past your 30 second summary or asked some follow up questions, walk them through your poster in more detail.
    - DON'T READ IT TO THEM!
  - A bottle of water is typically useful.

# Again

- Poster Presentation: **Best Project Prize!**
  - Dec 6th 1:30-3:30pm
  - Goodwin Hall Atrium
  - Print poster (or bunch of slides)
    - Fedex, Library, ECE support, CS support
  - Format:
    - Portrait, 2 feet (width) x 36 inches (height)
- Submit poster as PDF by Dec 6<sup>th</sup> 1:30pm
  - Makes up the final portion of your project grade
- If you are worried about your project, talk to me soon.

# Homework & Grading

- HW3 & HW4 should be graded this week
- Will release solutions this week as well



# Final Exam

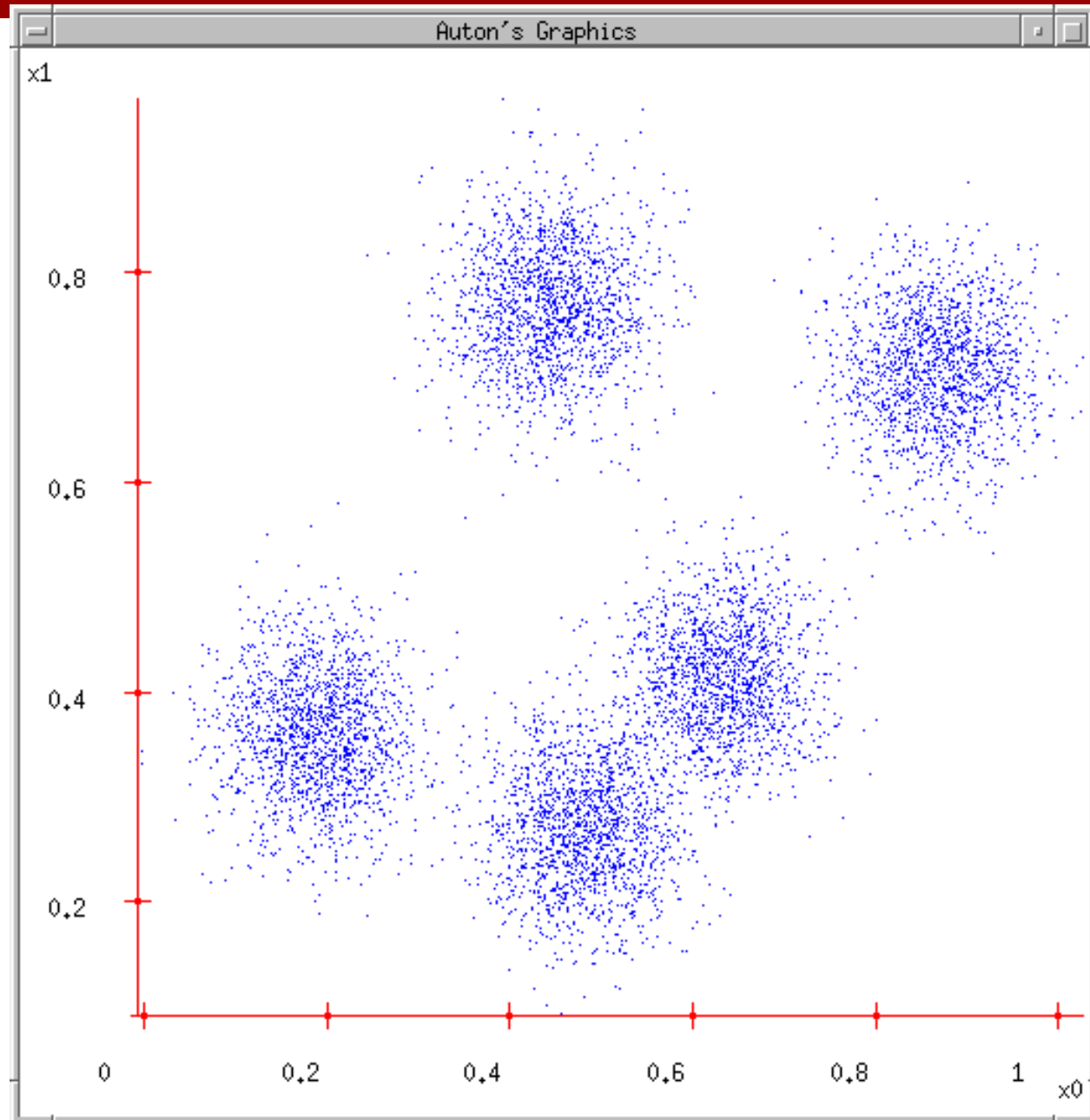
- Dec 14<sup>th</sup> in class (DURH 261); 2:05 - 4:05 pm
- Content:
  - Almost completely about material since the midterm
    - SVM, Neural Networks, Decision Trees, Ensemble Techniques, K-means, EM (**today**), Factor Analysis (**Thursday**)
  - True/False (explain your choice like last time)
  - Multiple Choice
  - Some 'Prove this'
  - Some 'What would happen with algorithm A on this dataset'

# One last thing

- SPOT surveys

# Recap of Last Time

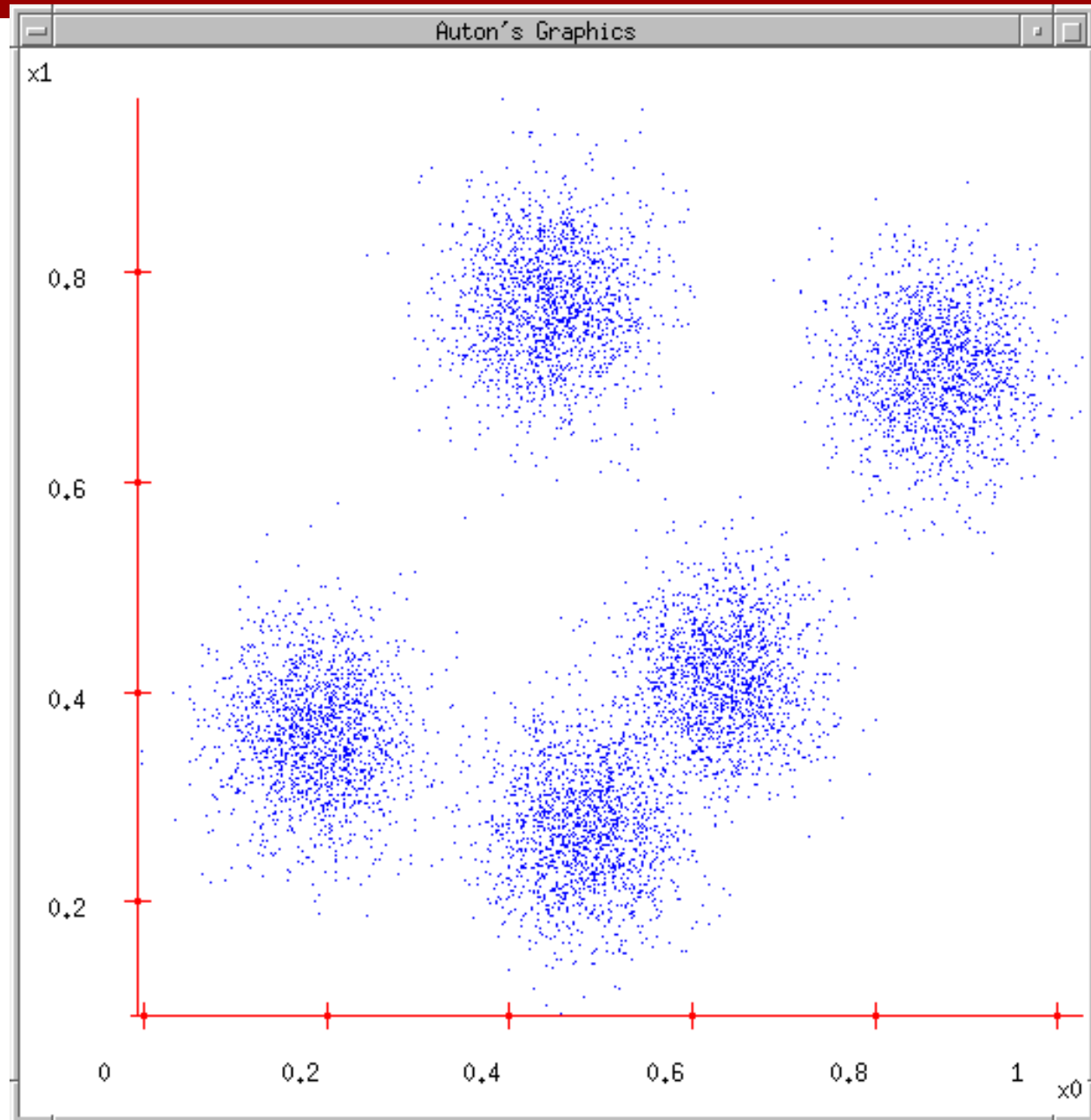
# Some Data





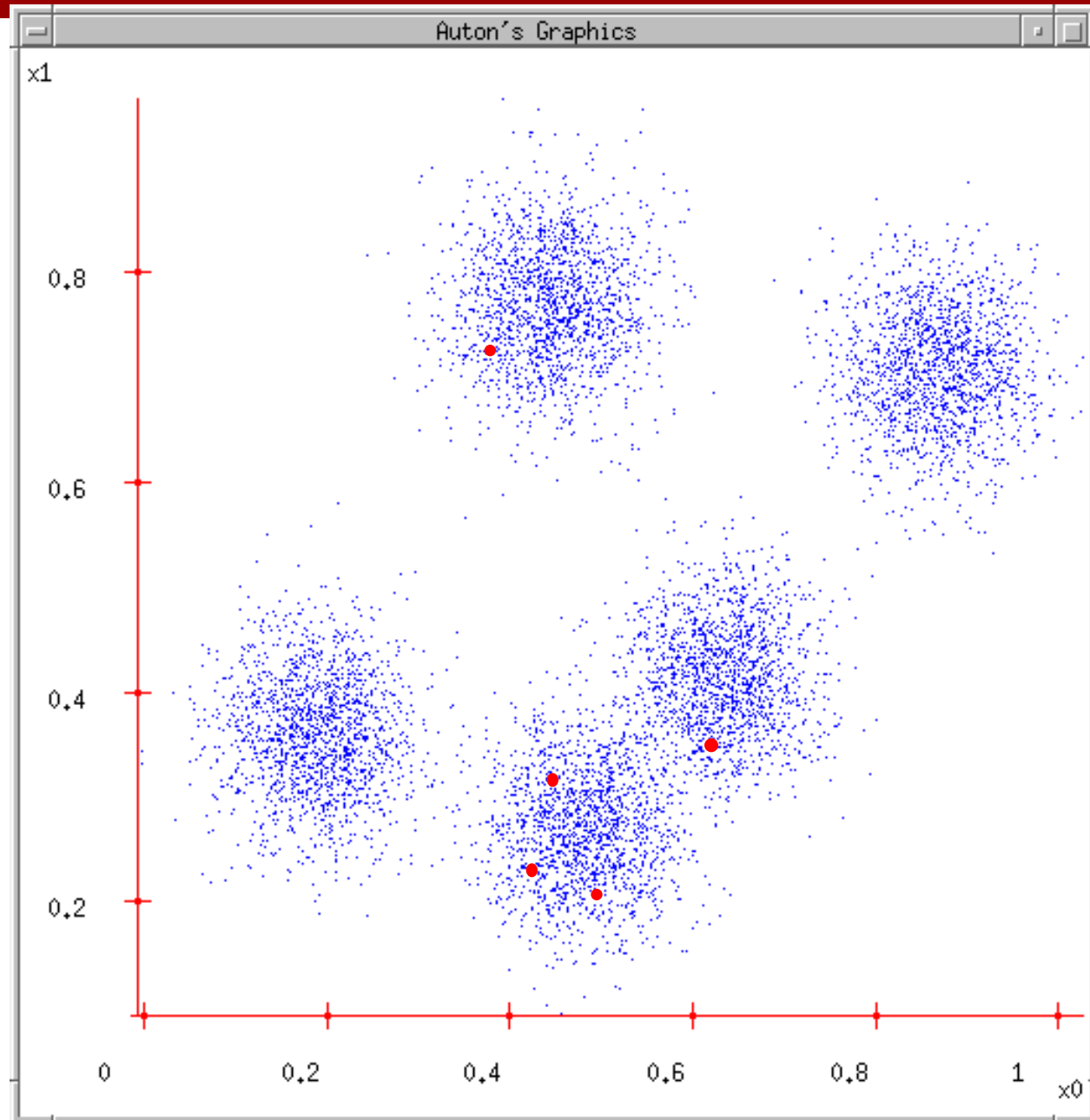
# K-means

1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )



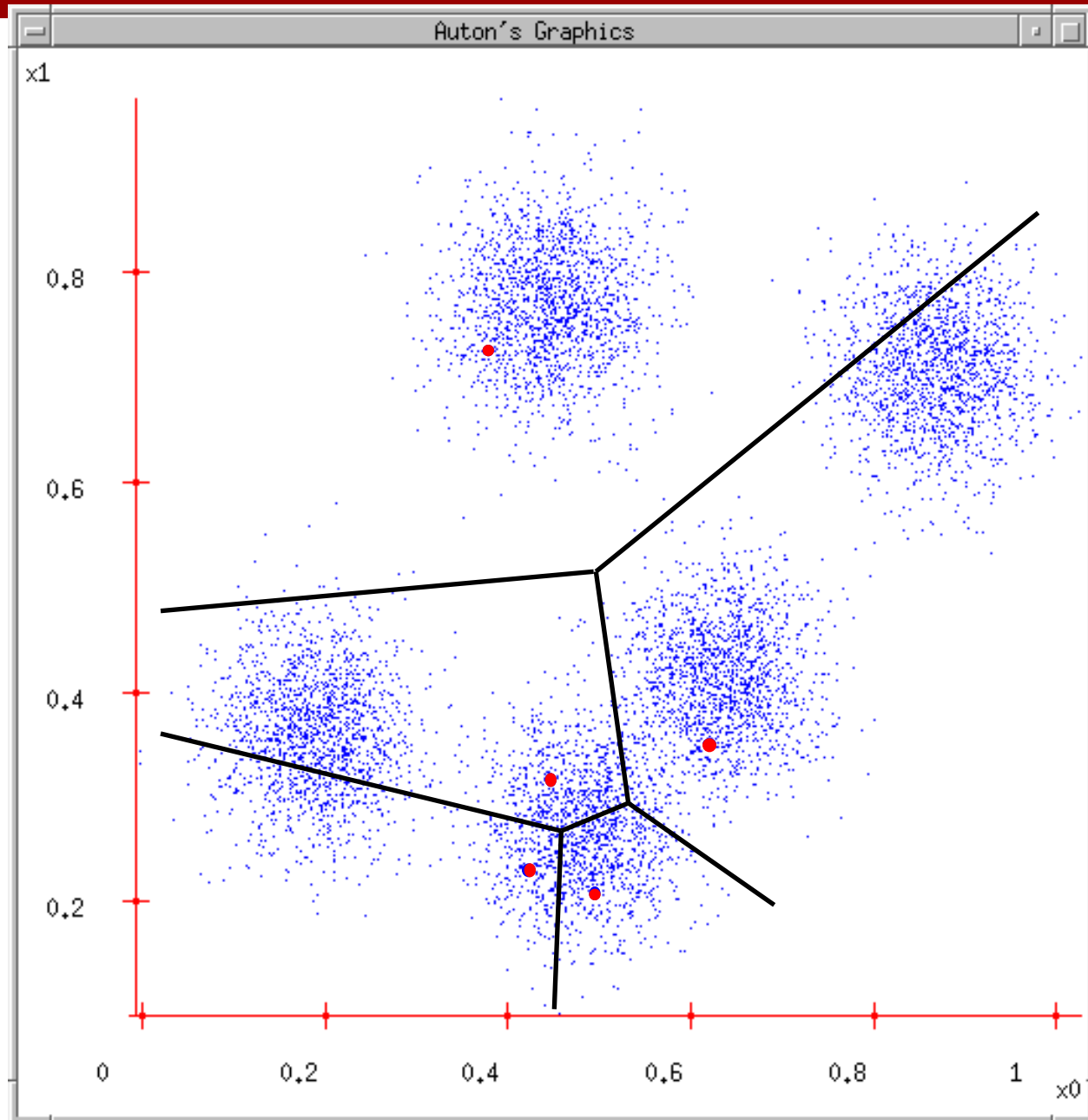
# K-means

1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )
2. Randomly guess  $k$  cluster Center locations



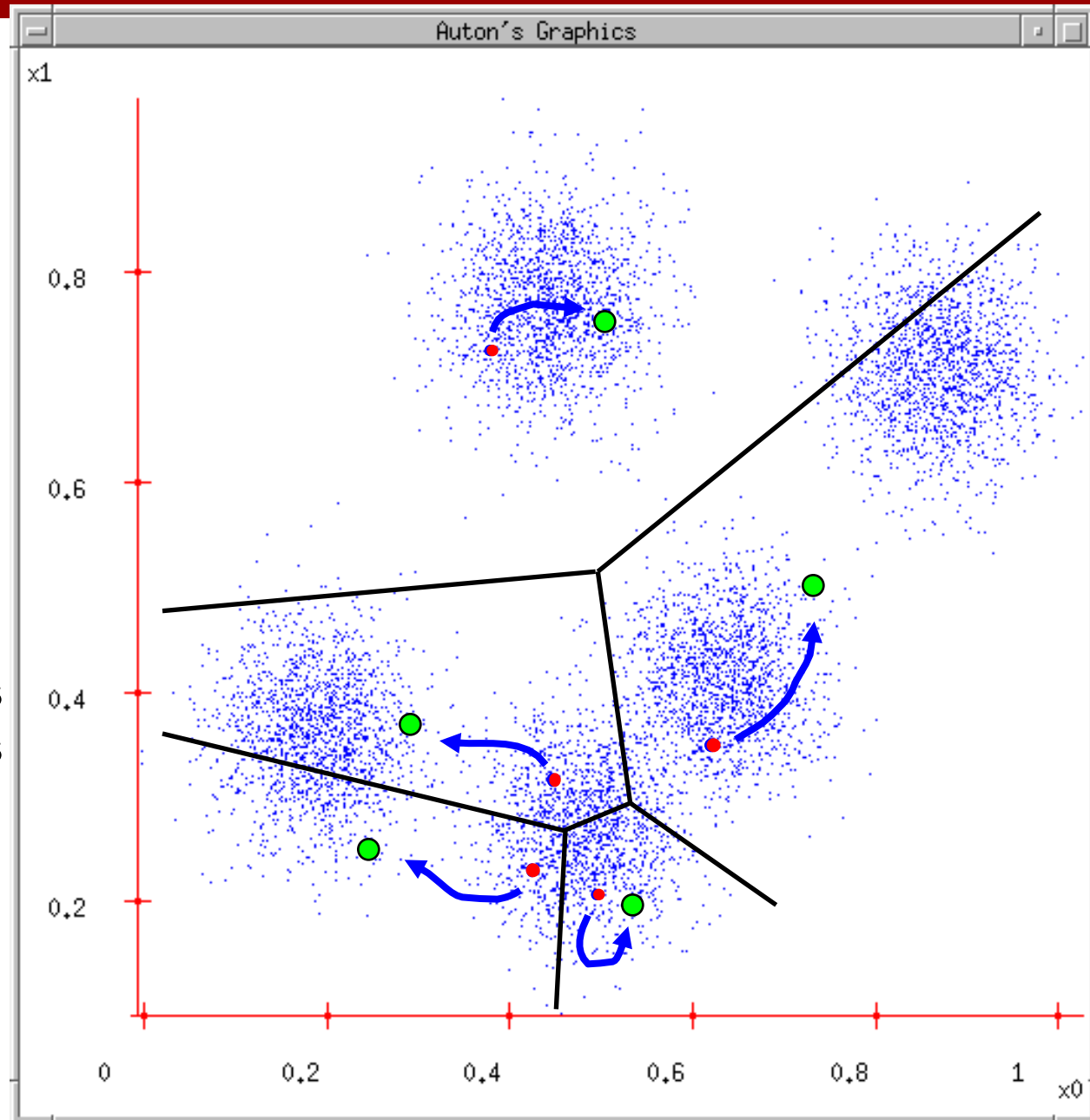
# K-means

1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )
2. Randomly guess  $k$  cluster Center locations
3. Each datapoint finds out which Center it's closest to. (Thus each Center "owns" a set of datapoints)



# K-means

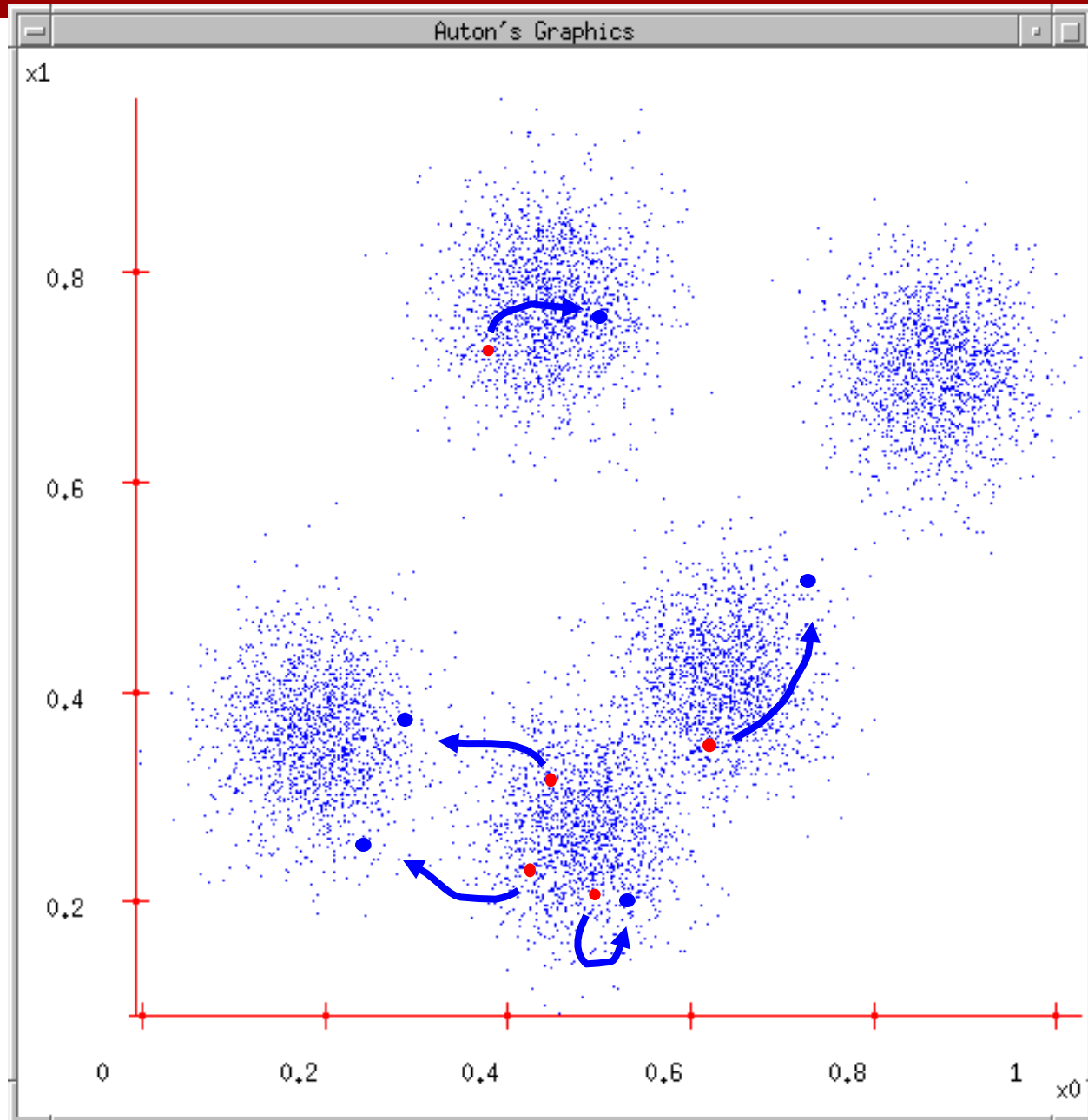
1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )
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3. Each datapoint finds out which Center it's closest to.
4. Each Center finds the centroid of the points it owns





# K-means

1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )
2. Randomly guess  $k$  cluster Center locations
3. Each datapoint finds out which Center it's closest to.
4. Each Center finds the centroid of the points it owns...
5. ...and jumps there
6. ...Repeat until terminated!



# K-means

- Randomly initialize  $k$  centers
  - $\mu^{(0)} = \mu_1^{(0)}, \dots, \mu_k^{(0)}$
- **Assign:**
  - Assign each point  $i \in \{1, \dots, n\}$  to nearest center:
  - $C(i) \leftarrow \underset{j}{\operatorname{argmin}} \|\mathbf{x}_i - \mu_j\|^2$
- **Recenter:**
  - $\mu_j$  becomes centroid of its points

# K-means as Co-ordinate Descent

- Optimize objective function:

$$\min_{\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_k} \min_{\mathbf{a}_1, \dots, \mathbf{a}_N} F(\boldsymbol{\mu}, \mathbf{a}) = \min_{\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_k} \min_{\mathbf{a}_1, \dots, \mathbf{a}_N} \sum_{i=1}^N \sum_{j=1}^k a_{ij} \|\mathbf{x}_i - \boldsymbol{\mu}_j\|^2$$

- Fix  $\boldsymbol{\mu}$ , optimize  $\mathbf{a}$  (or  $\mathbf{C}$ )

# K-means as Co-ordinate Descent

- Optimize objective function:

$$\min_{\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_k} \min_{\mathbf{a}_1, \dots, \mathbf{a}_N} F(\boldsymbol{\mu}, \mathbf{a}) = \min_{\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_k} \min_{\mathbf{a}_1, \dots, \mathbf{a}_N} \sum_{i=1}^N \sum_{j=1}^k a_{ij} \|\mathbf{x}_i - \boldsymbol{\mu}_j\|^2$$

- Fix  $\mathbf{a}$  (or  $\mathbf{C}$ ), optimize



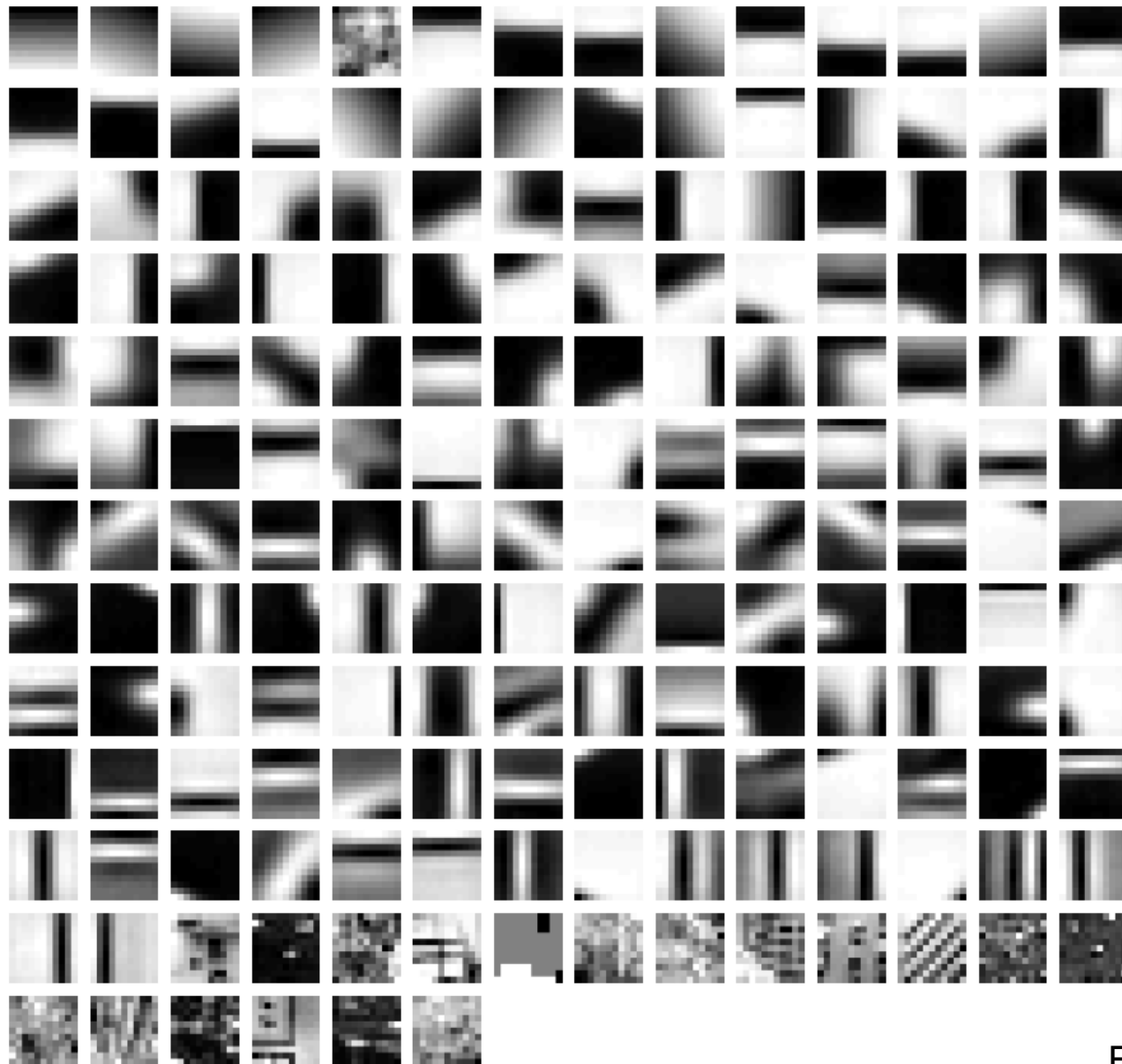
**Object**



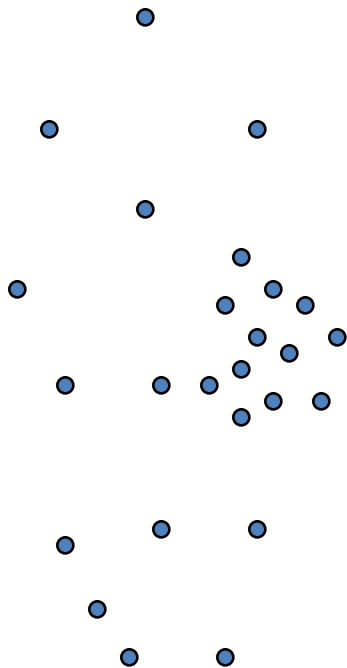
**Bag of 'words'**



# Clustered Image Patches

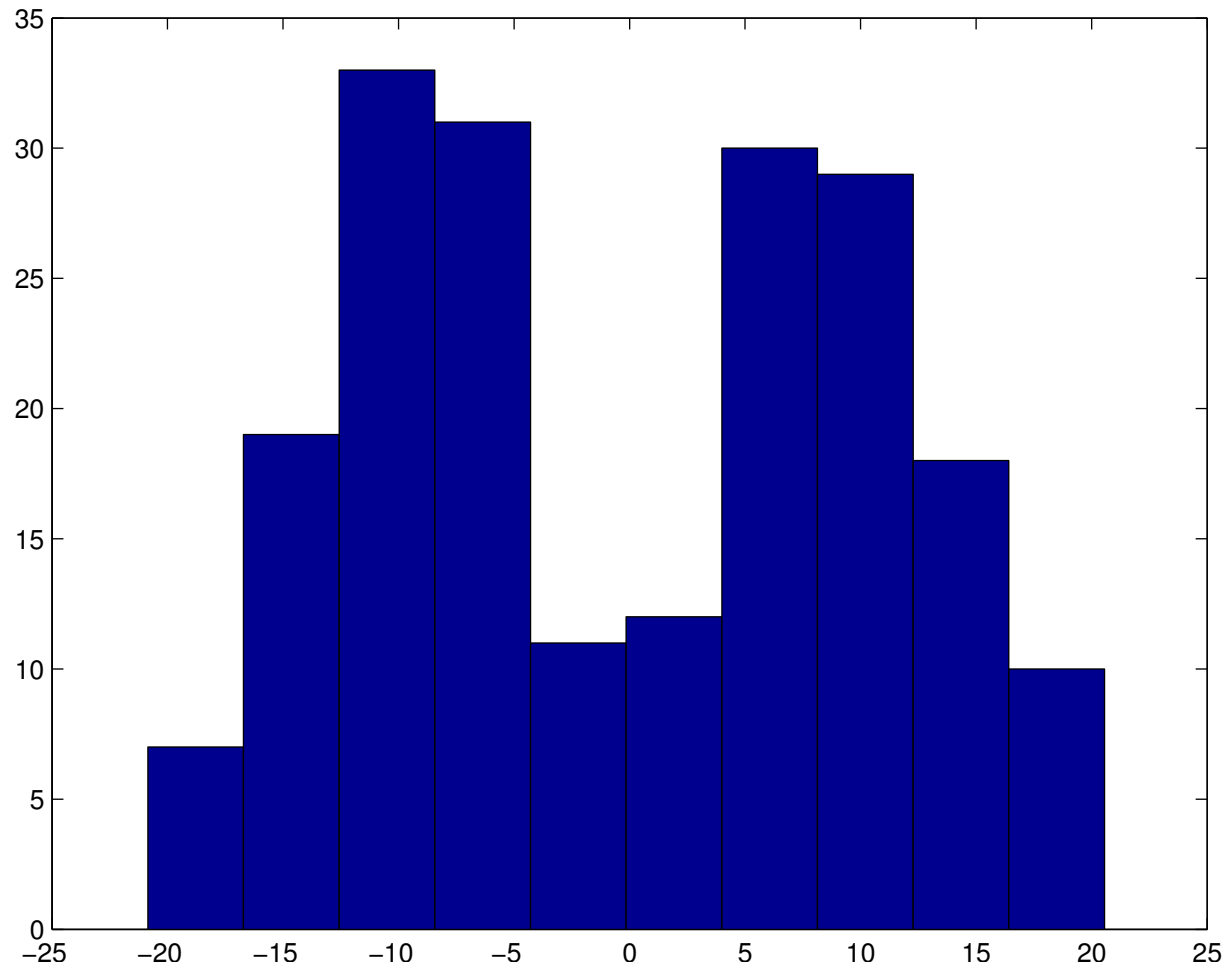


# (One) bad case for k-means

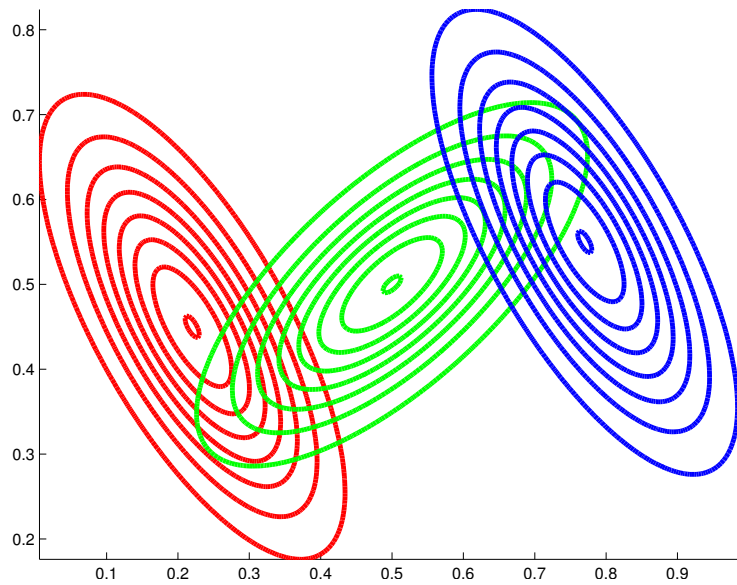


- Clusters may overlap
- Some clusters may be “wider” than others
- GMM to the rescue!

# GMM



# GMM



# K-means vs GMM

- K-Means
  - [http://home.deib.polimi.it/matteucc/Clustering/tutorial\\_html/AppletKM.html](http://home.deib.polimi.it/matteucc/Clustering/tutorial_html/AppletKM.html)
- GMM
  - <http://www.socr.ucla.edu/applets.dir/mixtureem.html>

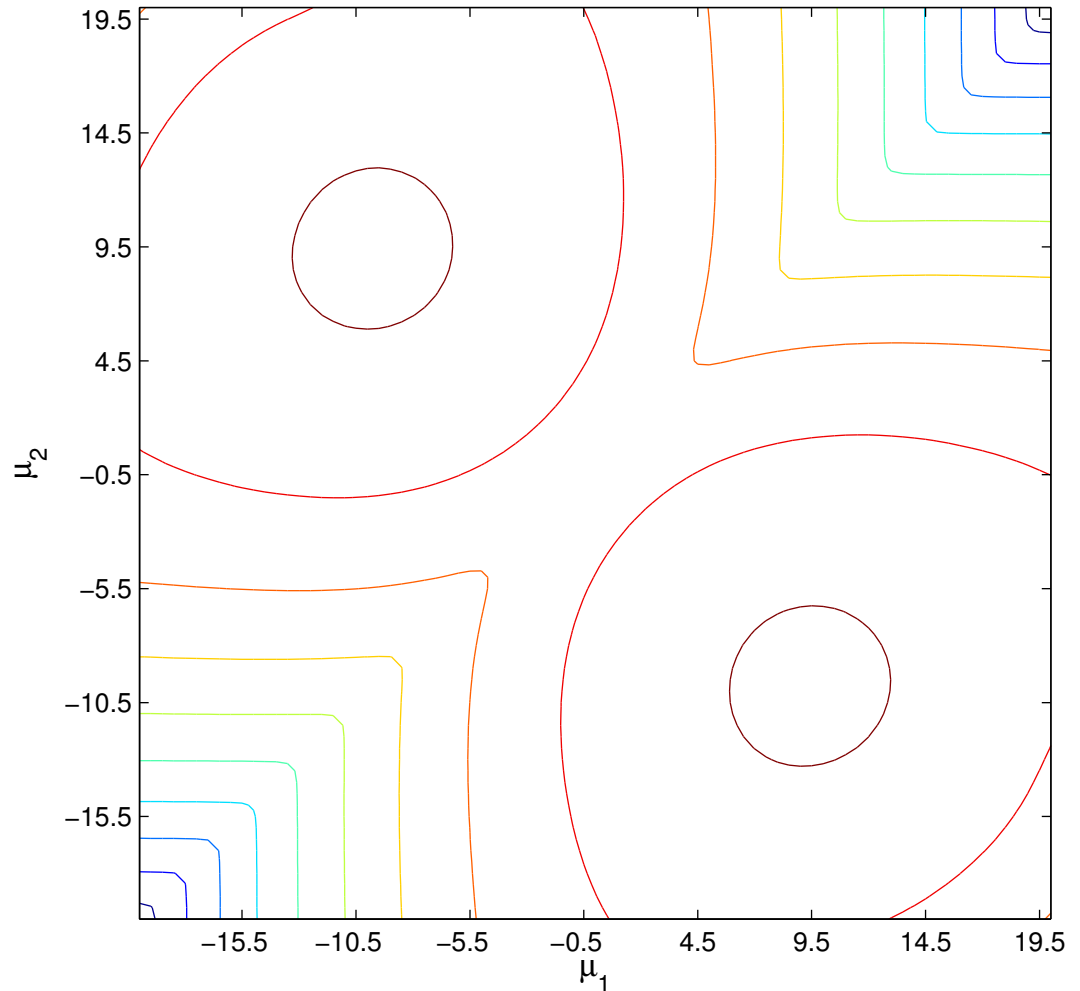
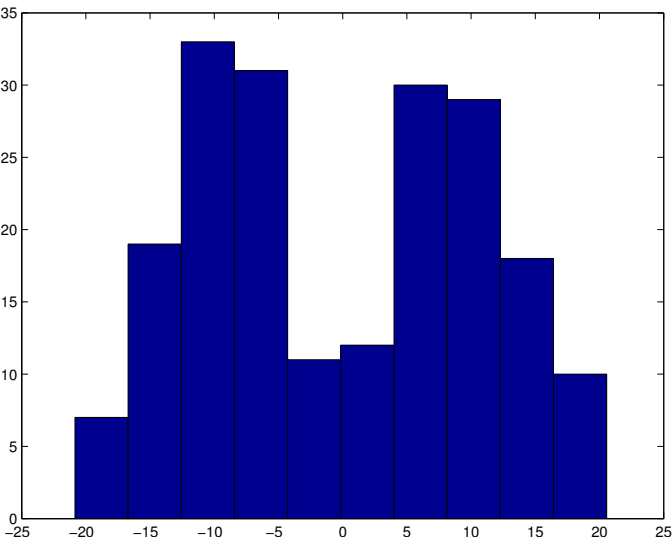
# Hidden Data Causes Problems #1

- Fully Observed (Log) Likelihood factorizes
- Marginal (Log) Likelihood doesn't factorize
- All parameters coupled!



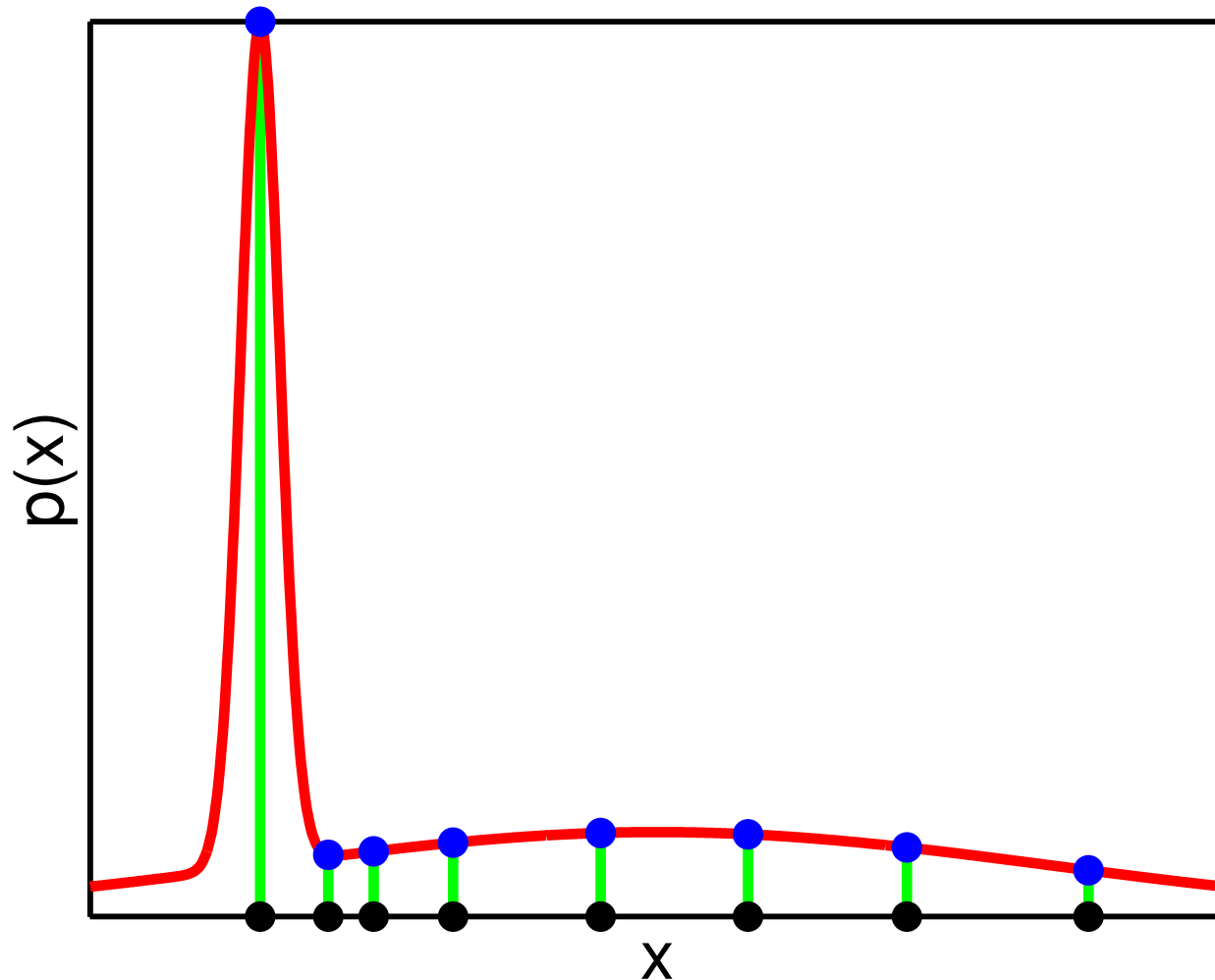
# Hidden Data Causes Problems #2

- Identifiability



# Hidden Data Causes Problems #3

- Likelihood has singularities if one Gaussian “collapses”



# Special case: spherical Gaussians and hard assignments

- If  $P(\mathbf{X}|Z=k)$  is spherical, with same  $\sigma^2$  for all classes:

$$P(\mathbf{x}_i | z = j) \propto \exp\left[-\frac{1}{2\sigma^2} \|\mathbf{x}_i - \mu_j\|^2\right]$$

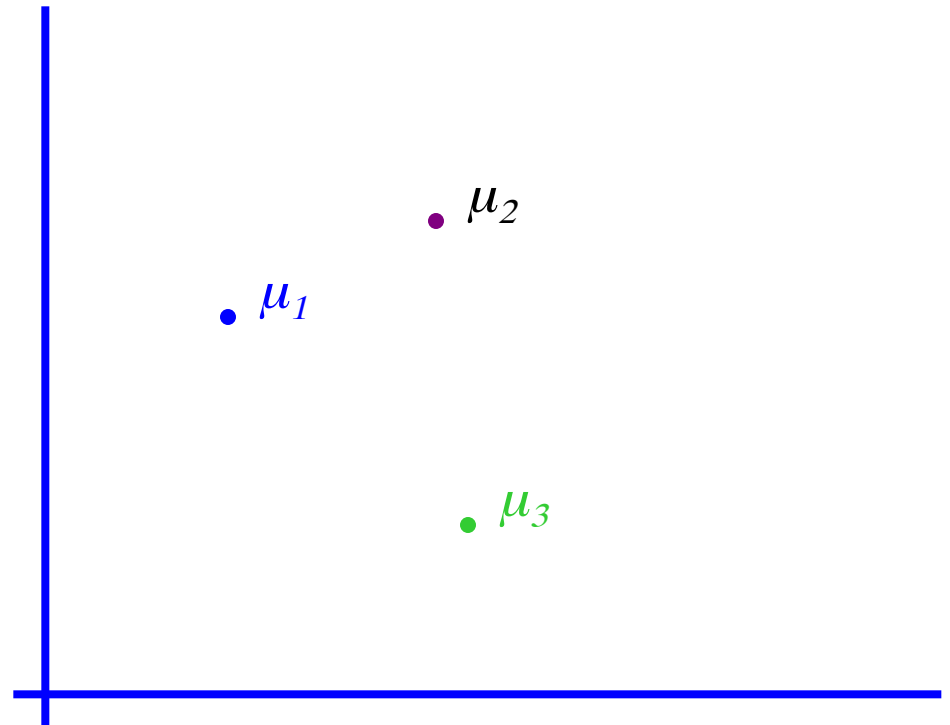
- If each  $\mathbf{x}_i$  belongs to one class  $C(i)$  (hard assignment), marginal likelihood:

$$\prod_{i=1}^N \sum_{j=1}^k P(\mathbf{x}_i, y = j) \propto \prod_{i=1}^N \exp\left[-\frac{1}{2\sigma^2} \|\mathbf{x}_i - \mu_{C(i)}\|^2\right]$$

- M(M)LE same as K-means!!!

# The K-means GMM assumption

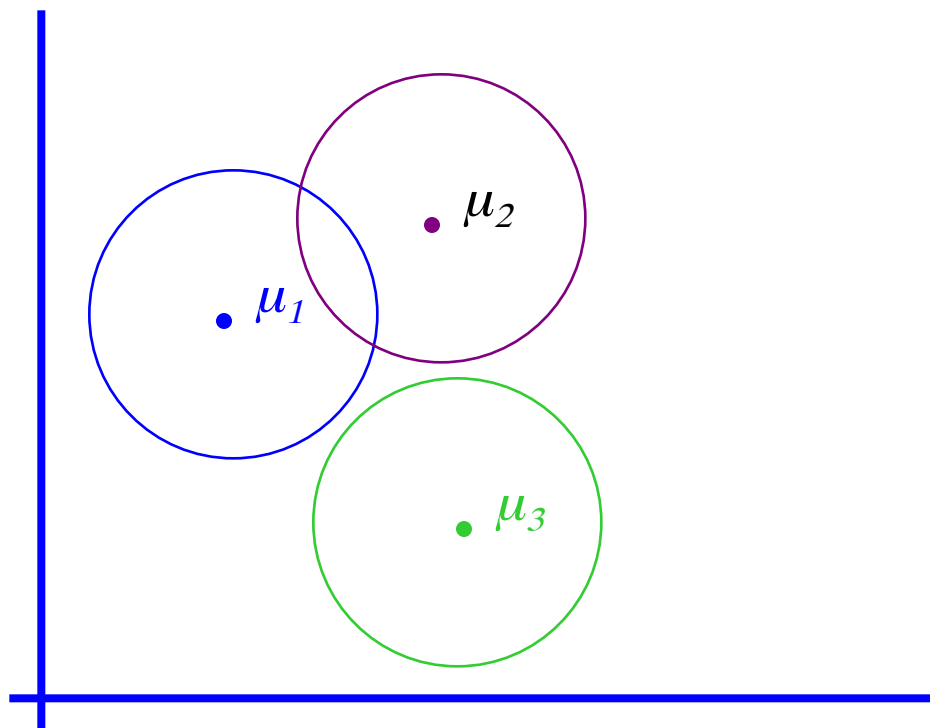
- There are  $k$  components
- Component  $i$  has an associated mean vector  $\mu_i$



# The K-means GMM assumption

- There are  $k$  components
- Component  $i$  has an associated mean vector  $\mu_i$
- Each component generates data from a Gaussian with mean  $m_i$  and covariance matrix  $\sigma^2 I$

Each data point is generated according to the following recipe:

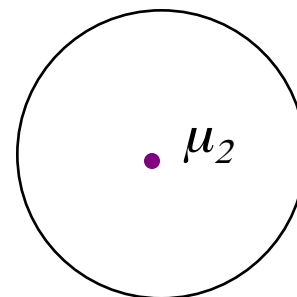


# The K-means GMM assumption

- There are  $k$  components
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Each data point is generated according to the following recipe:

1. Pick a component at random:  
Choose component  $i$  with probability  $P(y=i)$

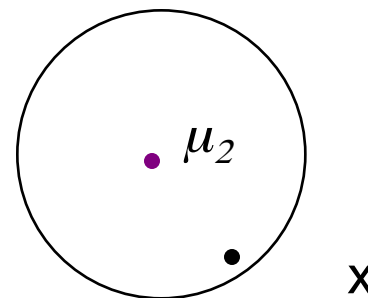


# The K-means GMM assumption

- There are  $k$  components
- Component  $i$  has an associated mean vector  $\mu_i$
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Each data point is generated according to the following recipe:

1. Pick a component at random:  
Choose component  $i$  with probability  $P(y=i)$
2. Datapoint  $\sim N(\mu_i, \sigma^2 I)$



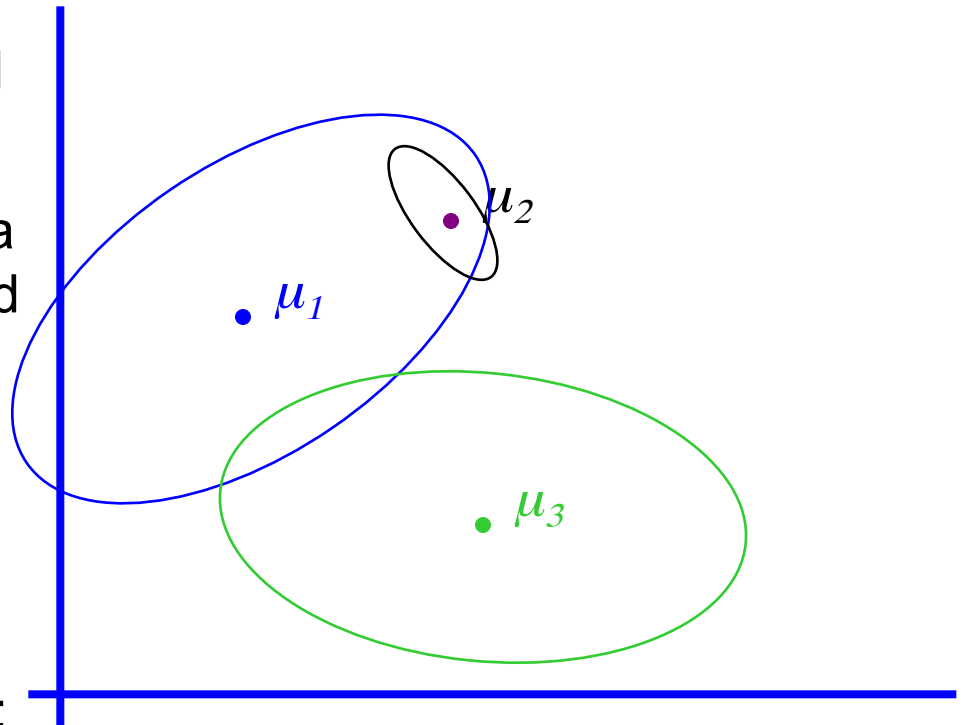


# The **General** GMM assumption

- There are  $k$  components
- Component  $i$  has an associated mean vector  $\mu_i$
- Each component generates data from a Gaussian with mean  $m_i$  and covariance matrix  $\Sigma_i$

Each data point is generated according to the following recipe:

1. Pick a component at random:  
Choose component  $i$  with probability  $P(y=i)$
2. Datapoint  $\sim N(\mu_i, \Sigma_i)$



# K-means vs GMM

- K-Means
  - [http://home.deib.polimi.it/matteucc/Clustering/tutorial\\_html/AppletKM.html](http://home.deib.polimi.it/matteucc/Clustering/tutorial_html/AppletKM.html)
- GMM
  - <http://www.socr.ucla.edu/applets.dir/mixtureem.html>

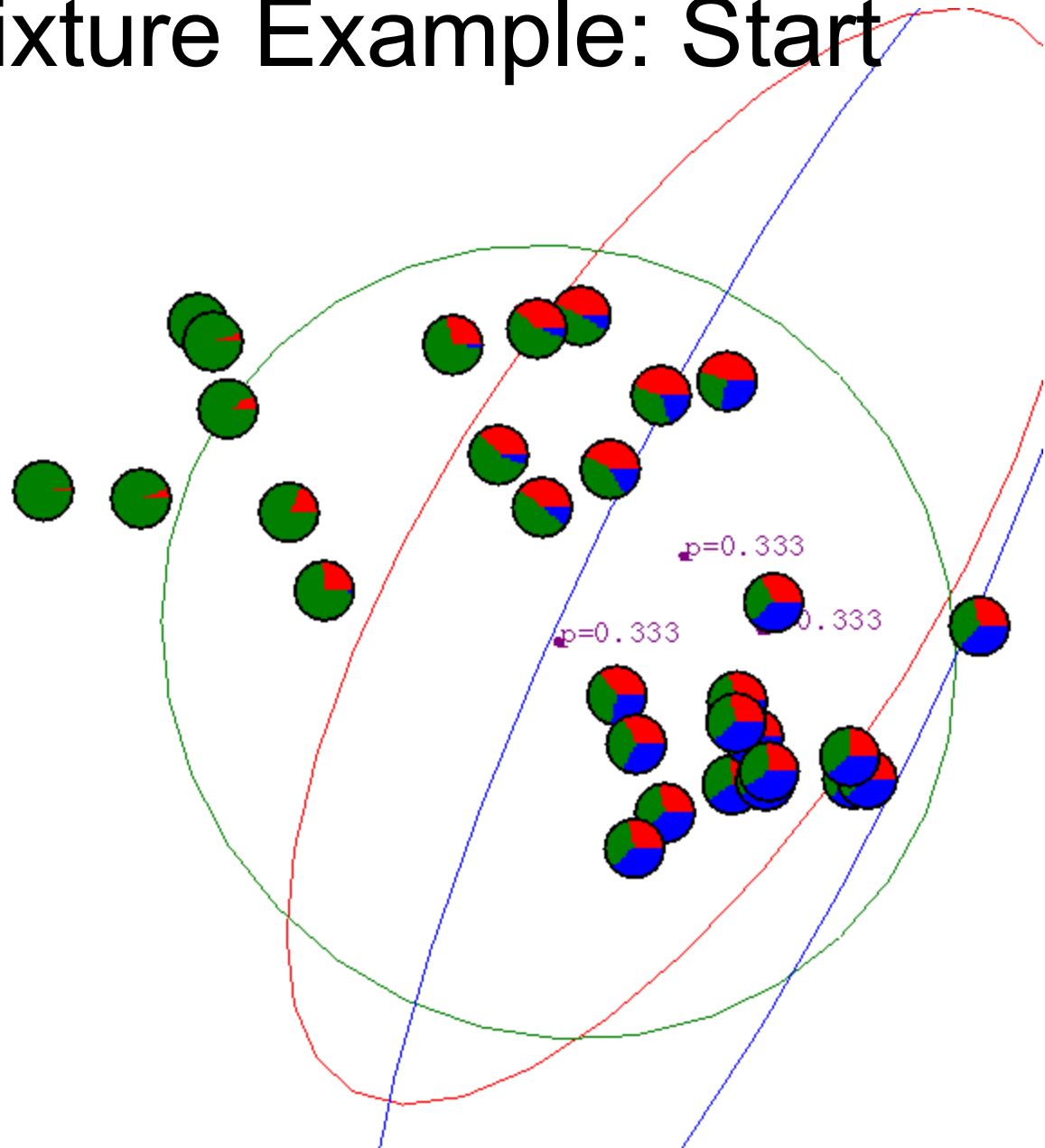
# EM

- Expectation Maximization [Dempster '77]
- Often looks like “soft” K-means
- Extremely general
- Extremely useful algorithm
  - Essentially THE goto algorithm for unsupervised learning
- Plan
  - EM for learning GMM parameters
  - EM for general unsupervised learning problems

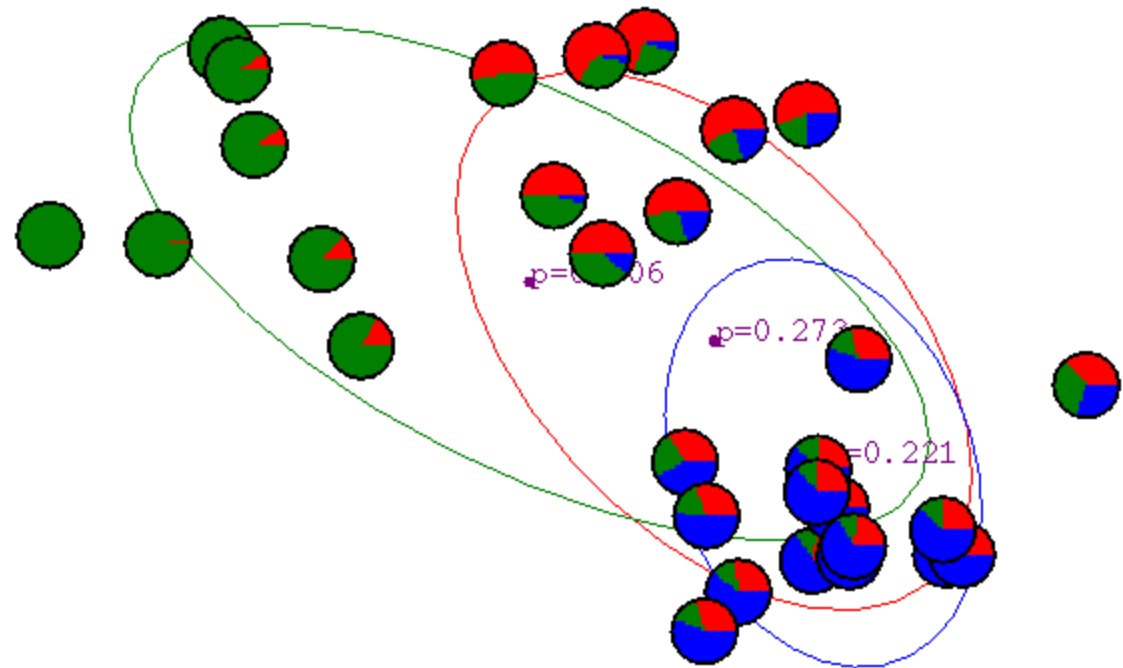
# EM for Learning GMMs

- Simple Update Rules
  - E-Step: estimate  $P(z_i = j \mid x_i)$
  - M-Step: maximize full likelihood weighted by posterior

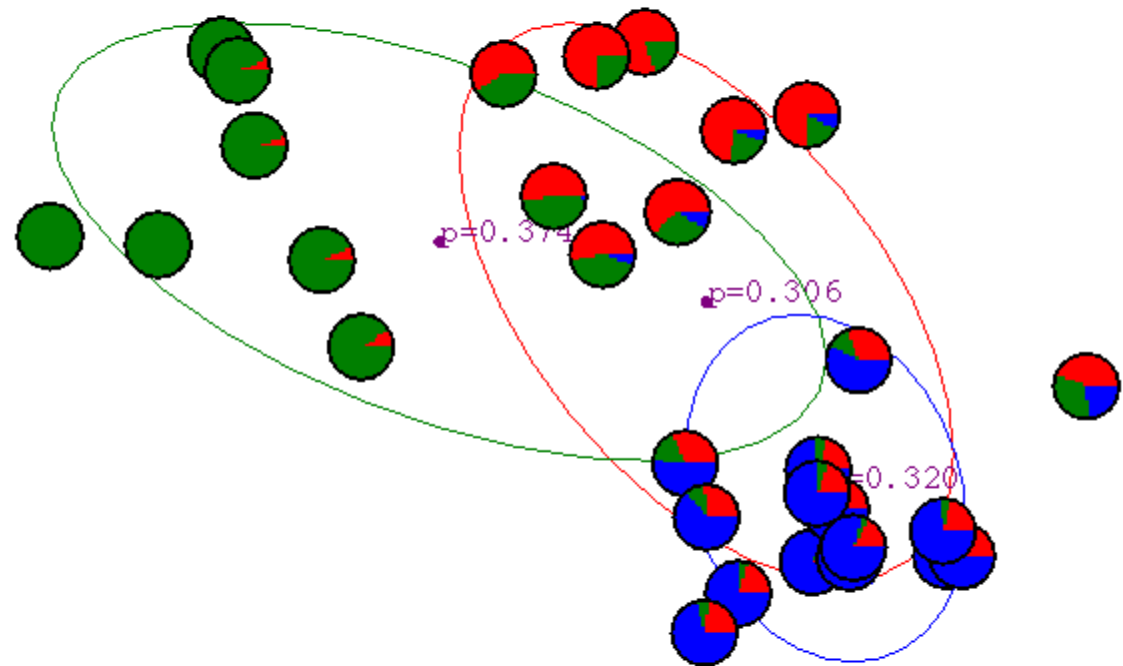
# Gaussian Mixture Example: Start



# After 1st iteration

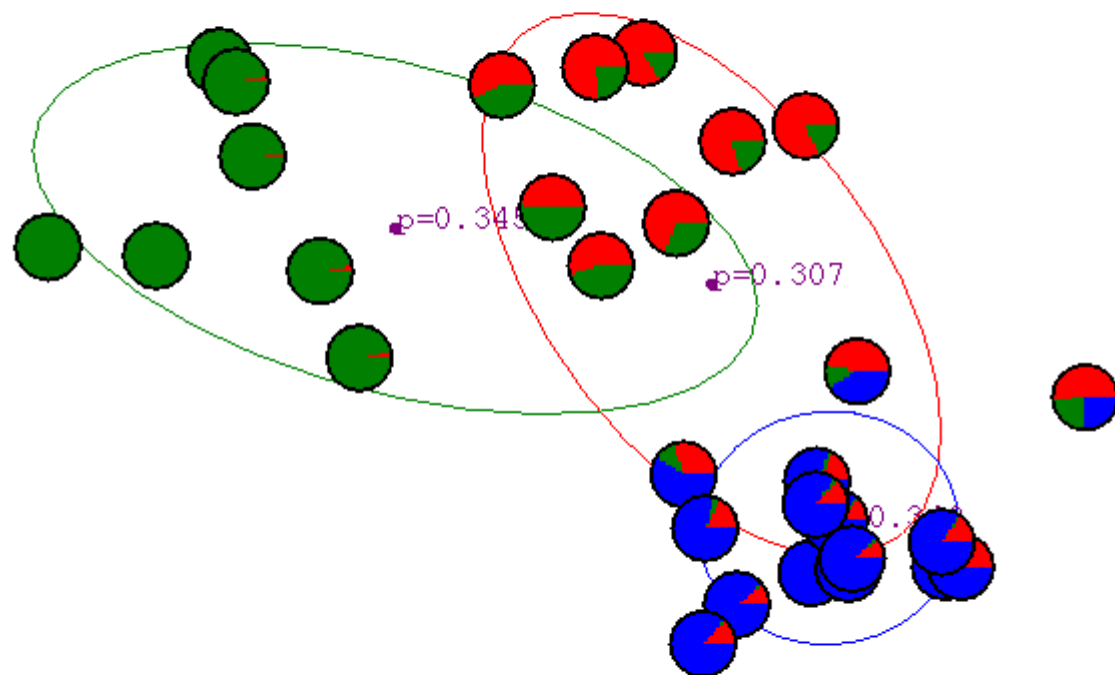


# After 2nd iteration

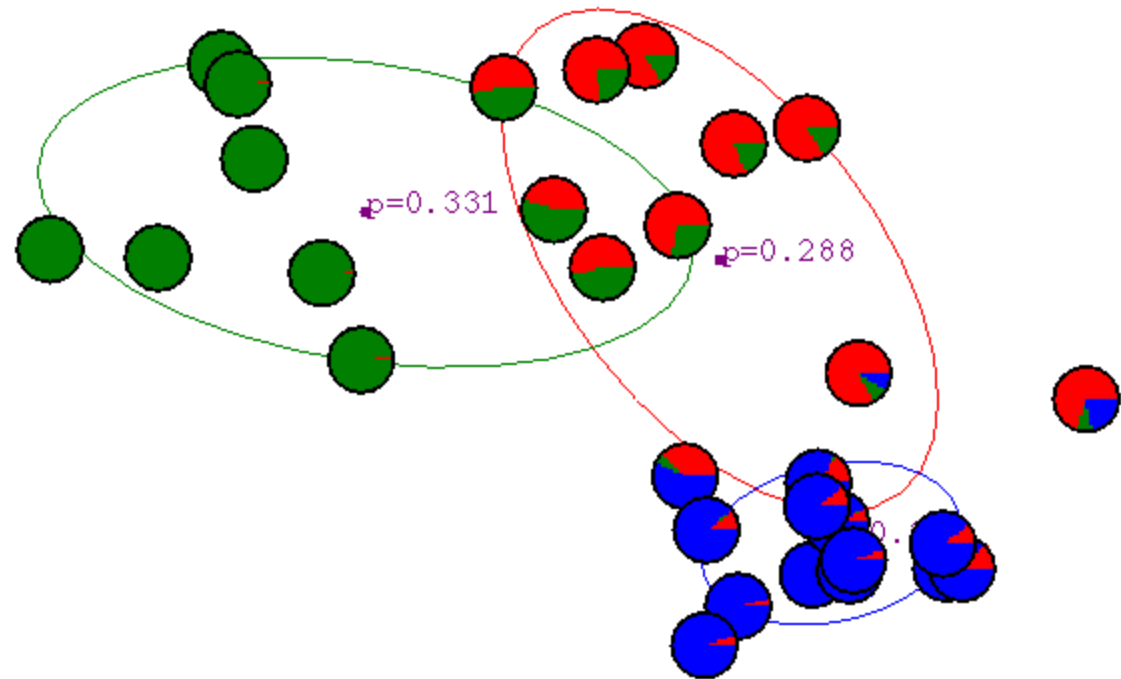




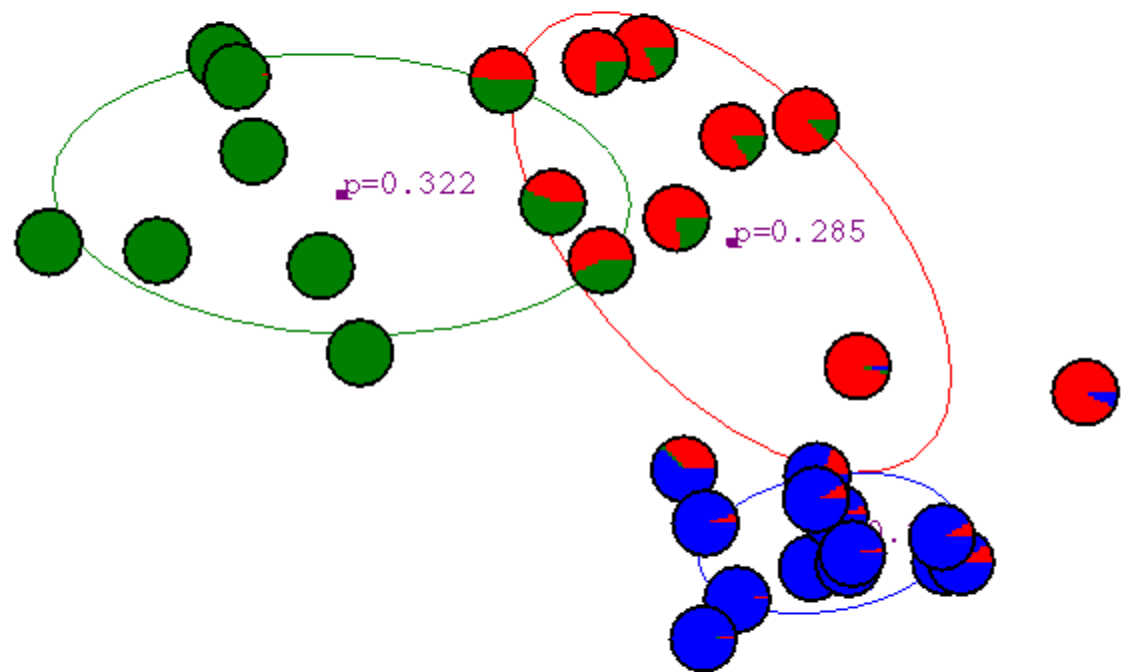
# After 3rd iteration



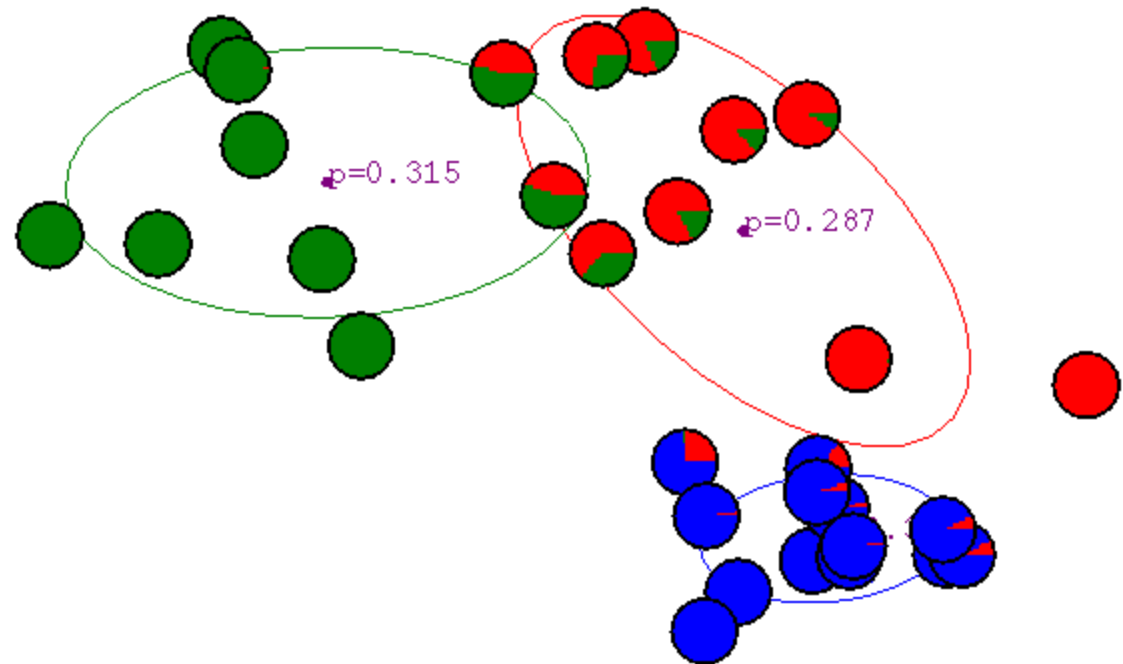
# After 4th iteration



# After 5th iteration



# After 6th iteration



# After 20th iteration

